# Results from a Prototype Chicane-Based Energy Spectrometer for a Linear Collider 

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#### Abstract

The International Linear Collider and other proposed high energy $\mathrm{e}^{+} \mathrm{e}^{-}$ machines aim to measure with unprecedented precision Standard Model quantities and new, not yet discovered phenomena. One of the main requirements for achieving this goal is a measurement of the incident beam energy with an uncertainty of $10^{-4}$ or less. This article presents the analysis of data from a protoype energy spectrometer commissioned in 2006-2007 in SLAC's End Station A beamline. The prototype was a 4-magnet chicane equipped with beam position monitors restoring the beam orbit through the chicane. An energy resolution close to $5 \cdot 10^{-4}$ was estimated, which, however, needs to be improved for a linear collider. We also report on the operational experience with the chicane-based spectrometer and suggest ways of improving its performance.


Keywords: Energy measurement, Energy Spectrometer, Cavity Beam

[^0]Position Monitor, BPM, End Station A, ESA, International Linear Collider, ILC

## 1. Introduction

The physics potential of the next TeV-energy Linear Collider depends greatly on precision energy measurements of the electron and positron beams at the interaction point (IP). Such measurements are mandatory in order to determine particle masses in high-rate processes. For example, measuring the top mass from a threshold scan to order of 100 MeV or measuring the Standard Model Higgs in direct reconstruction to about 50 MeV requires knowledge of the luminosity-weighted mean collision energy to a level of $1-2 \cdot 10^{-4}$ to avoid center-of-mass energy $(\sqrt{s})$ uncertainties from dominating the experimental results. Incoming beam energy $\left(E_{b}\right)$ measurements are a critical component to $\sqrt{s}$ determination as it sets the overall energy scale for the collision process.

The strategy proposed in the International Linear Collider (ILC) design [1] is to have redundant beam-based measurements capable to achieve a $10^{-4}$ relative precision on a single beam, which would be available in real time as a diagnostic tool to the operators. Also, physics reference channels such as $e^{+} e^{-} \rightarrow \mu^{+} \mu^{-} \gamma$ where the muons are resonant with the known Z-mass are expected to provide valuable cross-checks of the collision energy scale, but only long after the data have been recorded.

The primary method planned to perform $E_{b}$ measurements at the ILC is a non-invasive beam position monitor (BPM) based energy spectrometer similar to a setup used for callibrating the energy scale for the W-mass measurement at LEP-II [2]. At the ILC, however, the parameters of the spectrometer are tightly constrained to provide limited emittance dilution at the highest ILC energy of 500 GeV .

Initially, a 3-magnet chicane located upstream of the interaction point just after the energy collimators of the beam delivery system (BDS) was proposed [3]. But the baseline ILC spectrometer design uses two dipole magnets to produce a beam displacement $x$, while two more magnets return the beam to the nominal beam orbit (as in fig. 1). For such a chicane, the beam energy is then given by

$$
\begin{equation*}
E_{b}=\frac{c \cdot e \cdot L}{x} \int_{m a g n e t} B d l \tag{1}
\end{equation*}
$$

where $L$ is the distance between the first two magnets and $\int B d l$ the integral of the magnetic field in each magnet. The 4-magnet chicane avoids spurious beam displacement signals in the BPMs due to beam tilts, and thus systematic errors in $E_{b}$ measurements. For this reason, a 4-magnet spectrometer, which maintains the beam axially with respect to the axis of the cavity BPMs, seems preferable over a more conventional 3-magnet chicane. In both cases the magnetic field in the spectrometer chicane can be recorded and reversed for studying systematic effects without changing the beam direction downstream of the spectrometer.

A dispersion of 5 mm at the center of the chicane can be introduced routinely without a significant degradation of the beam emittance due to synchrotron radiation. When operating a fixed dispersion of 5 mm over the whole energy range, a BPM resolution better than $0.5 \mu \mathrm{~m}$ is needed. This resolution can be achieved with cavity BPMs [4]. Since the spectometer bending magnets need to operate at low fields when running the ILC at the Z-pole, the magnetic field measurement may not be accurate enough to provide the required level of precision. A significantly improved BPM resolution would, however, allow the magnets to be run at the same field for both the Z-pole and highest energy operation.

An absolute energy measurement requires that the beam orbit measurement is referenced to the orbit with no field applied. Unfortunately, the residual fields still have an impact on the beam orbit at a level that may affect the overall beam energy accuracy. There is an ongoing $R \& D$ program to determine how to perform accurate field mesurements for very low magnetic fields [5].

Some original energy resolution studies of the SLAC prototype 4-magnet chicane were presented by M. Viti in ref. [6]. His analysis used calibrated beam position readings but revealed that due to small differences between the magnets in the chicane the beam inclination also needs to be considered. It was soon realised that the same analysis could be extended by using complex BPM readings that contain the information on both the beam offset and inclination. This approach eliminates the need for position calibration of the BPMs, while the whole system could be calibrated by means of an energy scan.

In this publication we present the analysis based on that idea, estimate the resolution of the spectrometer to compare it with the result of $8.5 \cdot 10^{-4}$ measured in [6]. We also consider the impact of different systematics on the energy measurement in order to improve the resolution to below the $10^{-4}$ level in future experiments.

## 2. Test Beam Setup and Spectrometer Hardware Configuration

A protype test setup for a 4-magnet chicane was commissioned in 2006 (the T-474 experiment) and extented in 2007 (the T-491 experiment) in the End Station A (ESA) beamline at the SLAC National Accelerator Laboratory [7].

In our experiments the electron beam generated by the main Linear Accelerator at SLAC was transported to the ESA experimental area through the 300 m long transfer line A including bending and focussing magnets, and diagnostic instruments such as stripline and RF cavity BPMs, charge sensitive toroids, a synchrotron light monitor, profile screens and diodes. The SLAC linac was providing single bunches at 10 Hz and a nominal energy of 28.5 GeV , a bunch charge of $1.6 \cdot 10^{10}$ electrons, a bunch length of $500 \mu \mathrm{~m}$ and an energy spread of $0.15 \%$, i.e. with beam properties similar to the ILC expectations at the highest energy currently available for electrons.

These unique beam parameters allowed us to test the capabilities of the proposed spectrometer under realistic beam conditions. Two feedback systems were in place for the ESA beam: one for its position and one for the energy. The position feedback stabilised the beam position and angle using cavity BPMs and corrector magnets upstream of the ESA area. The energy feedback stabilised the energy controlling the phase of the klystrons, and thus the accelerating gradient, in one of the linac sections and was also used for offsetting the energy from the nominal in $\pm 100 \mathrm{MeV}$ range.

The setup, as schematically shown in Fig. 1, includes four bending magnets denoted as 3B1, 3B2, 3B3 and 3B4, forming a chicane in the horizontal plane and high-precision cavity BPMs upstream, downstream and in between the dipole magnets. Two of these (BPMs 4 and 7) in the middle of the chicane were instrumented with precision movers. When the magnets were turned on, these BPMs were mechanically moved to ensure the beam offset fits the dynamic range of the BPM electronics. These movers were also used for position calibrations. Horizontal positions of three monitors (BPMs 5, 4 and 7) were monitored with a Zygo interferometer [8].


Figure 1: Schematic representation of the prototype spectrometer in ESA.

The 10D37 magnets from the old SPEAR injection beamline, refurbished for the use in the chicane, are 37 " long, 10 " wide on the pole faces and have a 3 " gap. They were run in series from a single power supply to minimize relative drifts. The magnets were studied during a set of measurements in the SLAC Magnet Measurement Laboratory. Magnetic field maps of the vertical field component $B_{y}$ were taken using NMR and Hall probes, while each $\int B d l$ was measured using a flip coil, which was calibrated against a moving wire system. Stability and reproducibility were at the focus of these measurements. Details of the field measurements can be found in $[6,9,10]$.

In situ at ESA, two NMR probes wth different, but overlapping working ranges and initially also one Hall probe were installed in the first magnet 3B1, while one NMR probe was positioned in each of the other three magnets, so that field integral values could be monitored. In the test data runs, the nominal magnetic field was $0.117 \mathrm{~T} \cdot \mathrm{~m}$ which corresponds to a magnet operation at 150 A . The stray field outside the magnets in the middle of the chicane was monitored using two low-field fluxgate magnetometers. One was placed on the girder to obtain the horizontal $(x)$ and vertical ( $y$ ) field components and the other on the beam pipe measuring $y$-component only. Properties of the probes and the fluxgate monitors are summarized in fig. 2.
[discussion on NMR]
In order to measure the beam orbit, 8 cavity BPMs, all operating in the S-band of the RF, were installed. Three of them were SLAC prototype ILC BPMs $(3,4,5)$ using cylindrical cavities with $x$ - and $y$-waveguides for the dipole mode coupling and monopole mode suppression. Each of the


Figure 2: Magnetic field diagnistics in the spectrometer chicane.
five SLAC linac type BPMs (1, 2, 9, 10, and 11) consists of three cavities: two rectangular ones for $x$ and $y$ separately to avoid $x-y$ couplings, and one cylindical cavity to provide charge and phase information. BPM 7 was a dedicated ILC prototype designed and manufactured in the UK for the use in the spectrometer. Unfortunately, this monitor could not be used in the analysis due to manufacturing problems [11]. Details on the performance of the BPM system and information on the A-line configuration can be found in [4].

BPMs 12 and 24 are placed in the bending arc region of the A-line, where horizontal dispersion reaches several meters. For our experiment they were instrumented with the same high-sensitivity electronics as all other BPMs in the ESA beamline, so that the energy measurements in the A-line and in the chicane could be performed simultaniously and cross-checked against each other.

## 3. Performance of the Prototype Spectrometer

### 3.1. Reconstruction of the beam orbit in the middle of the chicane

As the chicane magnets bend the beam in the $x$-plane, we are mainly interested in the horizontal beam position and angle, and, unless specified otherwise, we talk about the $x$-coordinate throughout this section.

The offset of the beam trajectory in the middle of the chicane has to be measured with respect to the nominal orbit position reconstructed using BPMs outside of the chicane. In order to predict the readings of BPM 4 we took data from a run with zero-current in the magnets and selected a "quiet period", when neither the beam nor the hardware were manipulated. Data from a run with magnets on could also be used for relative measurements and would result in a better prediction, but due to the residual dispersion in the beamline beam positions before and in the middle of the chicane are correlated. For that reason, only data from a run with magnets off were used.

BPMs 9, 10 and 11 were not used for the prediction because the impact of the chicane on the beam orbit when magnets are on is not fully compensated due to the assymmetry of the chicane, and the beam offset in these BPMs is correlated with the energy.

Due to alignment errors, there is also a correlation between the vertical beam position and angle before the chicane and the horizontal beam position and angle in the mid-chicane. Therefore, both $x$ - and $y$-readings from the BPMs upstream of the chicane ( $\mathrm{x} 1, \mathrm{x} 2$, $\mathrm{x} 3, \mathrm{x} 5, \mathrm{y} 1, \mathrm{y} 2, \mathrm{y} 3$ and y 5 ) were used in the analysis.

In our system signals generated by the BPMs were digitized and stored in data files for each event, i.e. for each beam trigger. They are digitally converted to the baseband in the analysis [4]. A complex digital local oscillator signal allows to decode both the amplitude and the phase of the signal's phasor along the waveform. Sampled at a point close to the peak and normalized by the phasor from the reference cavity, the converted waveforms give the real, in-phase (I), value and the imaginary, quadrature (Q), value, which contain the information on the beam offset as well as the inclination.

In order to reconstruct the beam orbit in the mid-chicane, the I and Q values from BPM 4 are correlated to the I and Q values from the upstream BPMs. We applied the Singular Value Decomposition (SVD) method [12] to several thousands of readings. Inversion of the matrix of the measured I and Q values for the selected BPMs provides a vector of coefficients, which relate
the I's and Q's of each BPM to those of BPM 4 so that a prediction can be made:

$$
\begin{align*}
& I_{B P M 4}=\alpha_{0}+\sum_{i} \alpha_{i}^{(I)} \cdot I_{i}+\sum_{i} \alpha_{i}^{(Q)} \cdot Q_{i},  \tag{2}\\
& Q_{B P M 4}=\beta_{0}+\sum_{i} \beta_{i}^{(I)} \cdot I_{i}+\sum_{i} \beta_{i}^{(Q)} \cdot Q_{i}, \tag{3}
\end{align*}
$$

where $\alpha$ 's and $\beta$ 's are the SVD coefficients.
The difference between the predicted and the measured values is called residual. In our case, the RMS residual is the precision of the orbit prediction and the resolution of BPM 4 added in quadrature. It sets the limit on the spectrometer resolution. The measured and predicted values for I and Q are plotted against each other in fig. 3. The points in these plots lie around the $y=x$ solid lines, which means the prediction works correctly. The histograms in the bottom part of fig. 3 show the residuals, for both the I and Q values.

It is clear that the I and Q residuals for BPM 4 are small compared to the average I and Q values, but the results in fig. 3 are still hard to interpret quantitatively. In order to set a scale we used the mover scan data. During the mover scan BPM 4 was moved in 0.25 mm steps from -0.5 to +0.5 mm off the nominal position. The precision of the mover system is about $10 \mu \mathrm{~m}$, but the moves can also be observed by the interferometer with a sub-micrometer precision. Fig. 4 shows the scan data as well as the position residual, which was calculated for the data used in the SVD computations above. A position residual of $2.73 \mu \mathrm{~m}$ was estimated, which is close to the estimate in [6] $(2.3 \mu \mathrm{~m})$. The difference can be explained by softer applied cuts and a different minimization algorithm.

Assuming a 5 mm average beam offset in the middle of the chicane for magnets on, the $2.73 \mu \mathrm{~m}$ precision of the BPM system sets an energy resolution limit of $5.5 \cdot 10^{-4}$ for our spectrometer prototype.

The reader may be confused by this precision estimate comparing it to our earlier published value in ref. [4], which was close to $1 \mu \mathrm{~m}$. This is due to the fact that BPMs 9,10 and 11 had to be excluded from the analysis. BPM 4 used to be in the middle of the analysed system, in the "centre of gravity", while in the spectrometer studies it, unfortunately, ended up on the edge. Clearly, the precision of the orbit reconstruction at the BPM 4 position was affected.


Figure 3: BPM 4 readings predicded from other BPMs in the beamline: I predicted vs I measured (top left), Q predicted vs Q measured (top right), I prediction residual (bottom left), Q prediction residual (bottom right).


Figure 4: BPM 4 position for a horizontal mover scan (left), BPM 4 residual during a quiet period (right).

### 3.2. Estimate of the beam energy and scale correction

The I and Q readings predicted for BPM 4 by all other BPMs can be subtracted from the measured values and, when the magnets are on, provide
information on how the beam trajectory changes with the energy.
When the magnets are turned on, BPM 4 is also moved by a few mm in order to keep the beam offset within its dynamic range. This movement is observed by the precision Zygo interferometer. According to the interferometer, BPM 4 was moved by 5.0034 mm between our selected runs with magnets on and magnets off. Using the IQ rotation and scale from the mover scan, we can predict the changes of the I and Q values of BPM 4. An offset of 5.0034 mm results in $I_{0}=8784$ and $Q_{0}=4605$, which were added to the I and Q values from the energy scan after the predictions had been subtracted (fig. 5, top left).


Figure 5: Beam energy measurements: prediction subtracted Q vs I for BPM 4 (offset by $Q_{0}$ and $I_{0}$ to take into account the 5.0034 mm move), the "energy line" fits the measured points (top left), energy calibration plot for the spectrometer (top right), beam energy measured during the scan (bottom left), spectrometer noise measured off the energy line (bottom right).

Due to small differences between the magnets a small inclination of the beam orbit is introduced along with the offset in the middle of the chicane. But the measured points in the IQ plane still lie on a straight line as both
the offset and inclination scale with the energy. Fitting the measured data to a straight line going through the centre of coordinates, we can get the IQ rotation of this "energy" line. Energy readings for each point are then calculated as a projection of this point onto the energy line.

In order to get the energy scale, individual readings are averaged for each step of the energy scan and then fitted to a straight line (fig. 5, top right). The slope of this line gives the energy scale and the offset - the measured nominal energy. This procedure results in a nominal energy of about 32.6 GeV , while it was kept to within $1 \%$ off 28.5 GeV during the run. The fit has an uncertainty of $1.4 \mathrm{GeV}(4.3 \%)$ due to drifts during the scan and prevails the total error of the measurement, but still does not explain the difference. It can be attributed to the scale error of the energy feedback, meaning that the energy scan was actually performed in a range of $0.874 \cdot 200 \mathrm{MeV}=175 \mathrm{MeV}$, and the energy scale factor must be corrected accordingly.

The energy measured by BPM 4 during the scan is shown in fig. 5, bottom left. The measured fluctuations seem to be comparable with the energy scan step. In the following sections we use the data from the energy BPMs in order to separate the actual energy fluctuations from noise, and also include additional data acquired with the setup, such as interferometer and NMR readings, to refine this measurement and estimate the resolution of the spectrometer.

The last plot in fig. 5 (bottom right) shows the distribution of the offsets of the measured points from the fitted line, the RMS of the distribution was estimated to 10 MeV , or $3.5 \cdot 10^{-4}$. This value is an optimistic resolution estimate, as BPM readings in that plane mainly change due to the noise in the BPM system. It does not include the effect of the magnetic field fluctuations acting along the energy line. Indeed, our estimate using position data (sec.) was $5.5 \cdot 10^{-4}$.

### 3.3. Resolution of the energy BPMs

Similarly to spectrometer data, we measured the RMS residual between the fitted energy line and the measured points for the energy BPMs 12 and 24. In this case we could only do a relative energy measurement, as the field of the bending magnets in the A-line could not be turned off, but we were still able to calibrate them using the energy scan data and take into account the energy feedback scale correction.

The measured noise is equivalent to 0.36 MeV for BPM 12 and 2.0 MeV for BPM 24, or $1.3 \cdot 10^{-5}$ and $7.0 \cdot 10^{-5}$ respectively, at the nominal beam energy of 28.5 GeV . The values are different because BPM 12 had an additional 20 dB amplifier installed in its electronics chain in order to compensate for cable losses, which improved its sensitivity and reduced the effect of the noise and granularity introduced by the digitizers.

Again, these estimates only take into account the noise in the BPMs, and do not take into account many other effects such as the beam jitter and the changes of the fields in the magnets. In fig. 6 we compare the energy readings of BPMs 12 and 24 after the energy calibration. An RMS residual of $4.8 \mathrm{MeV}\left(1.7 \cdot 10^{-4}\right)$ was found, which is about twice bigger than the noise measurements combined in quadrature. This indicates that the resolution of the energy measurements of BPMs 12 and 24 is, in fact, not limited by the BPM noise alone. Nevertheless, BPMs 12 and 24 still allow for energy fluctuations to be measured to better than $1.7 \cdot 10^{-4}$, which is well below the expected spectrometer resolution.


Figure 6: Comparison of BPMs 12 and 24: BPM 24 energy measurement vs BPM 12 (left), residual between BPM 12 and 24 measurements (right).

### 3.4. Dipole magnets

An essential prerequisite for the operation of the spectrometer in a Linear Collider is that the beam position downstream of the chicane is not energy dependant, and the upstream beam path is restored downstream. In other words, the chicane has to be symmetric. In a 4-magnet chicane it is also beneficial to match the magnets in each pair producing a parallel translation of the beam (a "dogleg"), so that the inclination of the orbit with respect to the original is kept to a minimum.

Magnetic field measurements were performed in March 2007 to study the response of the chicane. Some results are shown in fig. 7. Here, the differences between the measured and nominal magnetic fields are plotted as a function of the nominal value for both negative and positive polarities.


Figure 7: Offsets between the measured and nominal magnetic fields as a function of the nominal value of the four magnets in ESA: Negative current (left); Positive current (right).

During these measurements the field of the magnet 3B1 was monitored with a Hall probe, whereas for the other magnets NMR probes were used. As can be seen, 3B1, 3B2 and 3B3 follow the same trend, with a difference of a few tenths of a mT between 3B2 and 3B3, while 3B1 is off by about 1 mT . 3B4 shows field values much closer to the nominal ones, because only for this magnet a more accurate relation between the current and the field (as given in [6]) was determined and used for the field settings. The differences in fig. 7 can be explained by the residual magnetic fields, which were estimated to be $0.2 \div 0.4 \mathrm{mT}$ (see [6]). They are expected to depend on the history of the magnets and on the properties of the core material (as the design and composition of steel cores could not be fully accounted for).

As a consequence, the trajectory of the beam had a small inclination in the middle of the chicane and was not fully restored downstream of the chicane, resulting in energy changes being converted into position variations in BPMs 9, 10 and 11.

Using the data from the upstream BPMs the nominal beam position in the downstream BPMs can be predicted. Considering, for example, BPM 9 measurements after subtraction of the downstream BPMs prediction, we can clearly recognize a step-like behaviour in energy during the scan (fig. 8). Note that, although the net-integral field applied to the beam by the chicane is
very small, BPM 9 is still able to resolve the energy changes due to its high resolution.


Figure 8: Energy measured by BPM 9 during the scan (left), IQ plot of the measured BPM 9 readings with the predicted readings subtraced (right). The fitted line shows the IQ rotation of the energy measurements.

### 3.5. Energy resolution of the spectrometer

The energy measured by the spectrometer can also be predicted by the energy BPMs 12 and 24. The residual, besides the resolutions of each BPM, depends on the fluctuations of the magnetic fields, mechanical vibrations, as well as drifts and other systematic effects and non-linearities.

We first compare the relative energy measured by BPM 4 with the measurements of BPM 12 (fig. 9, top). This results in a resolution of 24 MeV or $8.4 \cdot 10^{-4}$. As this is worse than the precision of the orbit reconstruction, we decided to look for correlations using additional data and applying the SVD method by starting again from BPM 12 (but this time the scale is corrected by SVD to better match BPM 4 readings which results to a lower residual) and then adding more data in the matrix to better reconstruct the spectrometer measurements and understand the systematics. Table 1 summarises the results together with the residuals calculated using the same coefficients for a quiet period when the magnets were on and nothing was changed in the system. Looking for consistent improvements of the residual, we can identify the main sources of systematic errors.

The biggest step in residual reduction is observed when the data from BPMs 9, 10 and 11 are included in the computation. As we know, BPMs 9,10 and 11 are sensitive to the energy, but also to the net-magnetic field


Figure 9: Energy resolution measurement: energy measured by BPM 12 and BPM 4 (top left), residual between BPM 12 and BPM 4 readings (top right), energy measurement predicted by BPMs 12, 24 and additional parameters and BPM 4 reading (bottom left), residual between the prediction and BPM 4 reading (bottom right).
of the chicane. Since our system did not provide bunch-to-bunch magnetic field measurements, only interpolated field data could be used. Inclusion of such data in the analysis did not provide a consistent improvement. It is therefore very likely that rapid field changes are encoded in the downstream BPM data, which might be the reason for residual improvements, when their data is added to the SVD matrix.

Some further improvement is also noted when the charge data, as measured by one of the reference cavities, is included in the analysis, even though all the BPM data were normalised by the charge. This is best explained by the fact that BPMs 12 and 24, although very sensitive to energy changes, were not centered to their operating ranges, and were running close to saturation.

Ultimately, in order to achieve an energy resolution approaching $10^{-5}$, one has to monitor the relative motion of the BPMs in the beamline. An

Table 1: Energy residuals calculated for BPM 4 including additional parameters. $\Delta \sigma / \sigma$ is the contributed uncertainty calculated as a difference of resolutions subtraced in quadrature.

| Data included | Residual, MeV |  | $\Delta \sigma / \sigma$, x10-4 |  |
| :--- | :---: | :---: | :---: | :---: |
|  | energy <br> scan | quiet <br> period | energy <br> scan | quiet <br> period |
| BPM 12 | 23.45 | 21.53 | - | - |
| BPMs 12, 24 | 23.08 | 21.64 | 1.5 | 0.8 (up) |
| BPMs 12, 24 and NMR | 22.67 | 22.62 | 1.5 | 2.3 (up) |
| BPMs 12, 24, NMR <br> and fluxgate | 22.67 | 22.62 | - | - |
| BPMs 12, 24, charge (q), <br> NMR and fluxgate | 20.52 | 19.68 | 3.4 | 3.9 |
| BPMs 12, 24, 9, 10, 11, <br> q, NMR and fluxgate | 15.86 | 15.26 | 4.6 | 4.4 |
| BPMs 12, 24, 9, 10, 11, <br> q, NMR, fluxgate and <br> interferometer | 15.68 | 14.60 | 0.8 | 1.6 |

interferometer, once well tuned, seems to be a reliable, fast and precision tool. But since the mechanical vibrations observed were in the region of a few hundred nanometers, the Zygo interferometer in our setup only provided a moderate improvement to the energy measurement.

The final result of these investigations is shown in the bottom part of fig. 9 . With additional data included, the prediction tracks the spectrometer measurement better than given in the plot above. The resolution was measured to $15.7 \mathrm{MeV}\left(5.5 \cdot 10^{-4}\right)$ for an energy scan and $14.6 \mathrm{MeV}\left(5.5 \cdot 10^{-4}\right)$ for a quiet period. These numbers are in a good agreement with the estimate for the precision of the orbit reconstruction of $5.5 \cdot 10^{-4}$, which means that the weighting of different systematics has been performed correctly.

## 3.6. $X$ to $Y$ coupling

Even though the spectrometer chicane operates in the horizontal plane, the energy scan is also traced in the vertical plane. Firstly, alignment errors generate a small bend in the vertical direction and, secondly, internal crosstalk between the $x$ - and $y$-couplers of the BPMs create spurious offset in y
by an offset in $x$.
In order to estimate the total cross-coupling between the $x$ - and $y$-planes we again consider the energy scan data in the run with magnets on, this time to predict the vertical beam position in BPM 4 using the SVD coefficients obtained from the run with magnets off. Clearly, as seen in fig. 10 (left), the energy scan is traced in the measured $y$-offset. Due to different sensitivities of the $x$ - and $y$-channels in BPM 4, we used mover scan data in both directions to get the position scale, which is used to normalise the raw energy. For that reason the energy is given in terms of mm in fig. 10. One should, however, keep in mind that an energy change generates both a different offset and an inclination in the mid-chicane.

The plot on the right-hand side in fig. 10 shows the correlation between the energy measured in both planes. From the inclination of the line fitting the data points a rotation of BPM 4 of almost $25^{\circ}$ is derived, or an $x-y$ isolation of about 7.6 dB . Even without tuning, BPMs usually provide an isolation of 20 dB , that means the cross-talk can not be explained solely by the cross-coupling of the signals. At the same time, the rotation is too large to be caused entirely by the alignment errors. This indicates that both effects take place. For the future, it is therefore important to minimize the cross-talk in the BPMs and eliminate fake offsets by careful alignment of the spectrometer elements.


Figure 10: Effect of the chicane on the vertical beam trajectory: energy scan traced by BPM 4 in y (left), energy data measured by BPM 4 in y vs $x$ (right). Position calibration was used to exclude the difference in sensitivities, hence, the energy is expressed in terms of the offset (mm).

## 4. Suggestions for future experiments

Clearly, any improvement of the BPM resolution would have a significant positive impact on both the relative and absolute energy measurement as it reduces the BPM uncertainties contributing to the overall measurement error.

Improvement of the internal $x-y$ isolation in the BPMs would also have a positive impact on the energy measurement as the uncertainty introduced by the signal cross-coupled from the orthogonal direction would be smaller.

Higher resolusion BPMs could also simplify the operation of the spectrometer. For a 1 mm dispersion, a resolution of 100 nm would allow for a $10^{-4}$ energy uncertainty. Currently, a dynamic range of about 80 dB can be achieved with cavity BPMs which allows a 1 mm offsets to be measured with no need to move the BPMs. Hardware improvements and better algorithms to treat the signals saturating the electronics [?] are expected to expand the dynamic range to 90 and even 100 dB . Hence, systematic effects associated with moving the BPMs to track the beam when the magnets are on can be avoided without compromising the performance.

Without the need to move the BPMs when the chicane is in operation, the BPMs are not required to be mounted on precision movers for calibration purposes either, although simpler movers may still be mandatory for alignment. A direct calibration of the spectrometer can be performed by changing the phase of the RF in some accelerating modules, as it was done in our ESA experiment. Another way of calibration is to change the magnetic field by a small but known amount and restore the energy scale from the orbit changes.

Working with I and Q values of the BPMs directly, we realised that even a 4-magnet chicane does not generate a pure beam offset in the middle because of small differences between the magnets. At the required level of precision the inclination still needs to be taken into account in the energy measurements. Additional correctors after the chicane may be necessary to restore the original beam orbit. Even more importantly, in a 4-magnet chicane two magnets contribute to the uncertainty of the energy measurement.

These arguments suggest to return to the original 3-magnet chicane design as discussed in [3] and shown in fig. 11, where the central magnet, the spectrometer magnet, is instrumented with probes and the other two help to preserve the initial beam trajectory, and therefore can be controlled individually. High-precision BPMs in between the magnets provide information on the bend of the beam, while BPMs upstream of the first magnet predict
the default trajectory downstream. In this case, the spectrometer magnet produces a combination of offset and angle in the BPMs downstream, but all measured data should still lie on one line in the IQ space as in our analysis, see section 3.2.

Instrumenting the anscillary magnets and extending the interferometer onto the up- and downstream BPMs would provide redundant energy measurement at a low increment on cost. While the overall resolution is not expected to become improved as the anscillary magnets operate at half of the B-field strength of the spectrometer magnet, some systematic effects can be a priori excluded due to the opposite bend. Furthermore, some systematic errors can be detected because the bending angle is different (???). Also, BPM triplets instead of dublets in between the magnets would also provide redundancy of beam orbit measurements and improve both the precision and accuracy of the spectrometer. [need to explain all this?]


Figure 11: A 3-magnet spectrometer chicane.

To predict the default trajectory in a 3-magnet spectrometer, the IQ space of the BPMs can be scanned by changing the beam deflection of the ancillary magnets, while the spectrometer magnet is off. An additional corrector magnet downstream of the chicane may be required to fully restore the beam orbit.

A precision interferometer will be required to achieve the $10^{-4}$ or better level of precision. This becomes critical for a reduced dispersion as the BPM resolution must be enhanced to 100 nm , since RMS vibrations measured at ESA were in the order of 300 nm for stationary BPMs and approached $1 \mu \mathrm{~m}$ for BPMs mounted on the movers. The Zygo interferometer showed a precision fulfilling the spectrometer requirements, but carefully beam trigger synchronised interferometer readings must be provided.

The resolution of the spectrometer also depends on the availability of bunch-by-bunch B-field data. The time resolution of the NMR probes is in the order of tens of milliseconds, which is sufficient for bunch train averaged measurements in a linear collider, but not for bunch-by-bunch operation. [can we do anything?]

## 5. Summary

The model-independent analysis of the data obtained with the prototype Linear Collider spectrometer based on a magnetic chicane revealed that a resolution of $5.5 \cdot 10^{-4}$ can be achieved by means of a BPM system with micrometer level precision for beam orbit measurements. An improved BPM resolution is the key factor for enhancing the energy resolution. But to achieve the $10^{-4}$ level, fast and reliable monitoring of the magnetic field and the relative motion of the BPMs in the horizontal plane are required.

BPM resolutions can be pushed to the 100 nm level and below, which allows to reduce the dispersion in the chicane. In this case, beam emittance degradation caused by the spectrometer would be significantly reduced. Improving the beam energy resolution even further for dynamic ranges large enough to accomodate beam offsets of a milimeter would allow to operate the chicane without BPM movers, and hence eliminates associated systematic errors. Current analysis techniques combined with hardware improvements will enhance the dynamic range of cavity BPMs beyond the current limit of approximately 80 dB so that a reduced dispersion of the spectrometer is possible.

Working with uncalibrated in-phase and quadrature BPM readings, one can ignore any beam tilt from the magnets in the middle of a 4-magnet chicane, as both the angle and offset follow the energy changes and the IQ readings produce a straight line in the IQ space. For simplicity reasons, a 3 -magnet chicane becomes a prefered configuration. In this case an energy calibration of the whole system becomes however mandatory which could replace BPM calibration. Hence, any reference to a well known physics constant, such as the Z-mass, or a complementary method to measure $E_{b}$, is important for both to correct the scale of relative measurements and to establish the offset for absolute energy measurements.

A thorough simulation of the spectrometer taking into account detailed features of its components would benefit to understand the device and, possibly, simplify the design currently proposed for the baseline of the next linear
collider. Once an electron test beam with suitable energy becomes available, the results of the simulations should be verified in a real life environment.

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