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## 1 Introduction

Here are some loose thoughts, especially the 3th and 4th item would need some feedback.

- When there is a lot of angular jitter, this will affect the energy resolution. We take the Bdl to be constant normally in the calculation, but due to the angle of incidence the particles take a different path through the magnets. Hence we will need to study this effect and learn how to correct for this : maybe calculate the change in $\int d l$ as a function of the angle in incidence.
- When calculating the energy one needs to subtract the projected beam position at the center of the chicane, ie to subtract the orbit in front of the chicane. I think it's best done just using a linear extrapolation since something SVD'ish doesn't really work I think. With the SVD method, one assumes that the BPM reading at center of the chicane is linearly dependant on the beam positions, but that is just where it is not, since it depends in addition on $1 / E_{b}$. The SVD method will therefore try to compensate for the dispersion at center of chicane by chaning the coefficients from the BPM readings with no dispersion. It therefore doesn't work very well. Works better to fit a line and extrapolate...
- Formula to calculate the energy in a 4 magnet chicane... From figure 1 it follows:


Figure 1: Diagram showing the calculation of the energy from the displacement at center of chicane. Note that the sketch is a bit misleading as in reality $L_{m}>L_{b} \ldots$

$$
\begin{equation*}
\theta_{1}=\arctan \frac{R}{L_{m}} \approx \frac{R}{L_{m}}=\frac{e c}{E_{b}} \cdot \int_{B_{1}} B_{1} d l \tag{1}
\end{equation*}
$$

Where $\theta_{1}$ is the bending angle after the first dipole, $L_{m}$ the distance between the first to bending magnets, $B_{1}$ the magnetic field of the first dipole and $E_{b}$ the beam energy. $R$ is the translation that the beam would have if the particles saw exactly the opposite Bdl through both of the magnets. The approximation $\arctan x \approx x$ is valid since we are dealing with very small angles, of the order of mrad. We note the integral $\int_{B_{1}}$ as the path integral of the particle through magnet $B 1$. Furthermore we have

$$
\begin{equation*}
\tan \left(\theta_{1}+\theta_{2}\right) \approx \theta_{1}+\theta_{2}=\frac{x-R}{L_{b}} \tag{2}
\end{equation*}
$$

$\theta_{2}$ is the deflection of the beam through the second bending magnet and $x$ the x offset measured in the BPMs at the center of the chicane. $L_{b}$ is the distance between the second bending magnet and the BPM at centre chicane. Note that $\theta_{1}$ and $\theta_{2}$ have opposite sign due to the reverse polarisation of the first and second bending magnets. We can further write equation 2 as

$$
\begin{align*}
\frac{e c}{E_{b}}\left(\int_{B_{1}} B_{1} d l+\int_{B_{2}} B_{2} d l\right) & =\frac{x}{L_{b}}-\frac{R}{L_{b}} \\
& =\frac{x}{L_{b}}-\frac{e c L_{m}}{E_{b} L_{b}} \cdot \int_{B_{1}} B_{1} d l \tag{3}
\end{align*}
$$

Or, alternatively

$$
\begin{equation*}
E_{b}=\frac{e c}{x}\left(\left(L_{m}+L_{b}\right) \cdot \int_{B_{1}} B_{1} d l+L_{b} \cdot \int_{B_{2}} B_{2} d l\right) \tag{4}
\end{equation*}
$$

- To estimate the absolute uncertainty, we can simply use standard error propagation. In the following, we will abbreviate $\int B_{i} d l$ to $B_{i}$,

$$
\begin{align*}
\sigma_{E_{b}}^{2} & =\frac{e^{2} c^{2}}{x^{2}} \cdot\left[\frac{\sigma_{x}^{2}}{x^{2}}\left(\left(L_{b}+L_{m}\right) \cdot B_{1}+L_{b} B_{2}\right)\right)^{2}  \tag{5}\\
& +\sigma_{L_{b}}^{2}\left(B_{1}+B_{2}\right)^{2}+\sigma_{L_{m}}^{2} B_{1}^{2} \\
& \left.+\sigma_{B_{1}}^{2}\left(L_{b}+L_{m}\right)^{2}+\sigma_{B_{2}}^{2} L_{b}^{2}\right]
\end{align*}
$$

An important remark to make here is that we consider $\sigma_{x}$ to be the total uncertainty on the offset determination at center of chicane. This includes both contributions from the BPM system, so implicitly, the number of BPMs and their individual resolutions as well as the mechanical stability of the mover system on which the center of chicane BPM system is mounted. At a later point we should make a detailed study of the total uncertainty on $x$ as a function of BPM system configuration. I believe this can only be done with a full chicane simulation as the orbit determination is probably quite sensitive to alignment errors of the individual BPMs. We will tackle this in the simulation chapter.
Also the incoming orbit here is assumed to have no incident angle or offset. As we should determine the incident orbit with the BPM system in front of the chicane and extrapolate on track level to the center of the chicane, this should not matter in first other. However, as pointed out in the items above, when the incident angle and offset changes, the particles will follow different paths through the magnets due to small inhomogeneities. Therefore we need to again have a full spectrometer simulation to address this.
I have taken equation 5 and produced a couple of plots starting with the ESA chicane parameters. These are shown in table 1. For the errors on the distances between the magnets and the second magnet and the BPM we have assumed an uncertainty of 0.5 mm . The $\int B d l$ values are obtained by integrating the magnetic fieldmaps as measured in the SLAC MMF by Sergey and Michele.
The plot showing the uncertainty calculation result is in figure 2 in the next section.

|  |  |  |
| :--- | :--- | :--- |
| Energy | $E_{b}$ | 28.5 GeV |
| Distance between two first magnets | $L_{m}$ | 4.014 m |
| Distance between second magnet and BPM station | $L_{b}$ | 2.263 m |
| Alignment error along the beam line (z) | $\sigma_{L}$ | 0.5 mm |
| Integrated field in first magnet | $B_{1}$ | $-0.118214 \mathrm{T.m}$ |
| Integrated field in second magnet | $B_{2}$ | $0.125249 \mathrm{~T} . \mathrm{m}$ |
| Relative uncertainty on the integrated field | $\frac{\Delta B}{B}$ | $5.0 \mathrm{e}-5$ |

Table 1: The ESA parameters for the chicane

## 2 Energy resolution estimates

In this section we discuss some results based upon equation 5 .

## 3 For the ESA setup

Figure 2 shows a very rough estimation of the total uncertainty on the energy. The values for the parameters that are kept constant in each plot are shown in their respective plots by the dash-dotted vertical lines. They correspond to the numbers in table 1.

I'll try to simulate this system with the spectrometer chicane simulation and study these relations with real fieldmaps and alignment errors...

## 4 Extrapolation for the ILC

## 5 The spectrometer simulation

In this section we describe the full spectrometer simulation program which we have used to perform these studies.

### 5.1 Description of the Geant 4 simulation program

## PUT IN SOME STUFF ON THE SPECTROMETER SIMULATION PROGRAM

### 5.2 System simulation results for ESA

### 5.3 Extrapolation for the ILC



Figure 2: Estimation of the uncertainty of the energy measurement with a 4 magnet chicane starting from the ESA layout parameters.

